LECTURE NOTES: CHAPTERS 1 & 2 REVIEW

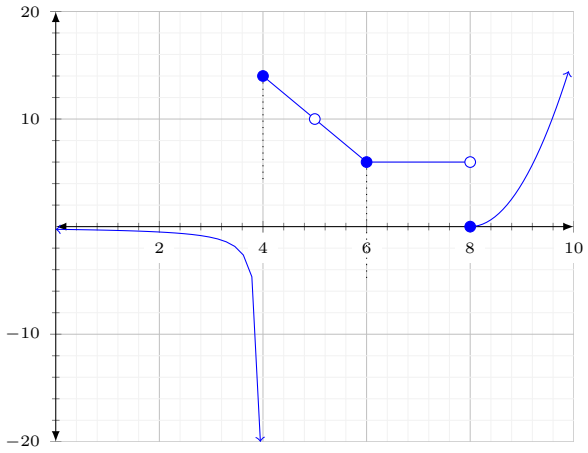
PRACTICE PROBLEMS:

1. Describe several ways in which a limit can fail to exists. Illustrate with sketches.

2. Describe what it means for a function $f(x)$ to be continuous at $x = a$ and several ways in which a function $f(x)$ can fail to be continuous at $x = a$. Illustrate with sketches.

3. Describe what it means for a function $f(x)$ to be differentiable at $x = a$. Illustrate with sketches differentiable and non-differentiable examples.

4. Use the graph of $f(x)$ below to answer the following questions.



(a) Assuming the arrows on the graph indicate a continued curve in that direction, make an educated guess at the domain of the function $f(x)$.

(b) Find all x-values in the domain of $f(x)$ for which $f(x)$

i. fails to be continuous.

ii. fails to be differentiable.

(c) Evaluate the following limits or explain why they do not exist.

(i) $\lim_{x\to 4^-} f(x) =$

(ii) $\lim_{x\to 4^+} f(x) =$

(iii) $\lim_{x\to 4} f(x) =$

(iv) $\lim_{x\to 5} f(x) =$

(v) $\lim_{x\to 6} f(x) =$

(vi) $\lim_{x\to 7} f(x) =$

(vi) $\lim_{x\to 8} f(x) =$

(vii) $\lim_{x\to 8^-} f(x) =$

5. (a) What does the Squeeze Theorem say? You may want to include a picture with your explanation.

(b) Use the Squeeze Theorem to show $\lim_{x\to 0^+} \sqrt{x}e^{\sin(\pi/x)} = 0$.

6. (a) What does the Intermediate Value Theorem say? You may want to include a picture with your explanation.

(b) Use the Intermediate Value Theorem to show $\ln x = x - 5$ has a solution. (Hint: Show there is a solution in the interval $[1, e^5]$.)

7. (a) Given a function $f(x)$, how do you determine whether or not its graph has any horizontal asymptotes?

(b) Given a function $f(x)$, how do you determine whether or not its graph has any vertical asymptotes? Give an example of a function $f(x)$ and x-value, c, such that the denominator of $f(x)$ is zero when $x = c$ but $f(x)$ has no vertical asymptote at $x = c$.

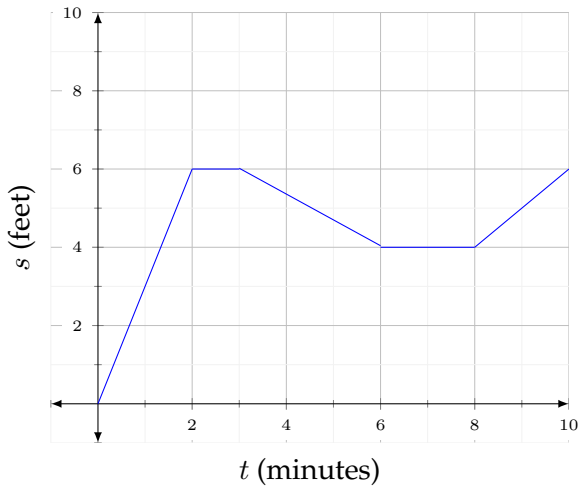
(c) Find the horizontal and vertical asymptotes (if any) of the graph of $f(x) = \frac{2x^2}{3x^2+2x-1}$.

8. Find the limit or show that it does not exist. In each case, write in your own words, what (if anything) your answers indicate about the graph of the given function.

(a) $\lim_{x \to -\infty} \dfrac{2 - x}{3x^2 - x} =$

(b) $\lim_{x \to \infty} [\ln(1 + x^2) - \ln(1 + x)] =$

9. A particle starts by moving to the right along a horizontal line; the graph of its position function is shown in the figure on the below.



(a) At what times is the particle moving to the right?

(b) At what times is the particle moving to the left?

(c) At what times is the particle standing still?

(d) Sketch a graph of the velocity of the particle.